AP Calculus AB Name Heinl 2016

Lessons 3-3, 3-5 & 3-9: *Rules for Differentiation, Part 2* Date

Learning Goal:

- *I can use rules of differentiation to calculate derivatives of polynomials, rational functions,* $y = e^x$ *and* $y = \ln x$*, and trigonometric functions.*

Let's talk about the derivatives of trigonometric functions!

1. In your calculator, in the window $[-2\pi, -2\pi]$ by $[-2, 2]$ graph $f_1(x) = \sin x$ and $f_2(x) = \frac{d}{dx}\sin x$.

   What do you think is the derivative of $y = \sin x$? Cosine!

   $\frac{d}{dx}\sin x = \cos x$

2. In your calculator, in the window $[-2\pi, -2\pi]$ by $[-2, 2]$ graph $f_1(x) = \cos x$ and $f_2(x) = \frac{d}{dx}\cos x$.

   What do you think is the derivative of $y = \cos x$? This one will take a bit more thinking . . .

   $\frac{d}{dx}\cos x = -\sin x$

3. Give the derivative of the sine and cosine functions above, and the fact that $\tan x = \frac{\sin x}{\cos x}$, use the quotient rule to derive the derivatives for $y = \tan x$ and $y = \cot x$.

$y = \tan x = \frac{\sin x}{\cos x}$

$$\frac{dy}{dx} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{(\cos x)^2}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$\frac{d}{dx}\tan x = \sec^2 x$

$y = \cot x = \frac{\cos x}{\sin x}$

$$\frac{dy}{dx} = \frac{\sin x(-\sin x) - \cos x(\cos x)}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{(\sin x)^2}$$

$$= \frac{-1(\sin^2 x + \cos^2 x)}{(\sin x)^2}$$

$$= -\frac{1}{(\sin x)^2} = -\csc^2 x$$

$\frac{d}{dx}\cot x = -\csc^2 x$

OVER →

Derivative of Trigonometric Functions

You need to know or be able to derive these!!

$(\sin x)' = \cos x$

$\cot x$ $(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc x)' = -\csc x \cot x$

Derivative of the "co" function is the negative complement of the primary function

$\sec x = \frac{1}{\cos x}$

$(\sec x)' = \frac{\cos x(0) - 1(-\sin x)}{(\cos x)^2} = \frac{\sin x}{(\cos x)^2} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$

Practice – Find $\frac{dy}{dx}$.

1. $y = 2\sin x - \tan x$

$\frac{dy}{dx} = 2\cos x - \sec^2 x$

2. $y = \frac{\cos x}{1 + \sin x}$

$\frac{dy}{dx} = \frac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)^2}$

$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$

$= \frac{-1(\sin x + \sin^2 x + \cos^2 x)}{(1+\sin x)^2}$

$= -\frac{1\cancel{(\sin x + 1)}}{(1+\sin x)^{\cancel{2}}} = -\frac{1}{1+\sin x}$

3. $y = 3x + x\tan x$

$\frac{dy}{dx} = 3 + 1 \cdot \tan x + x\sec^2 x$

$= 3 + \tan x + x\sec^2 x$

4. Show analytically (NOT using a graph) that the graph of $y = \sec x$ has a horizontal tangent line at $x = 0$

$\frac{dy}{dx} = \sec x \tan x$ $\qquad m_{T.L.} = 0$

$0 = \sec x \qquad 0 = \tan x$

$0 = \frac{1}{\cos x} \qquad x = \ldots -\pi, \boxed{0}\ \pi, 2\pi, \ldots$

Not possible

$0 \neq \sec x$

5. Given $y = 1 + \cos x$, find the equation of the tangent line at $x = \frac{\pi}{2}$. Sketch the graph and the tangent line.

$y' = -\sin x$

$y'(\frac{\pi}{2}) = -\sin\frac{\pi}{2} = -1 \Rightarrow$ slope of T.L.

$y(\frac{\pi}{2}) = 1 + \cos\frac{\pi}{2} = 1 \qquad (\frac{\pi}{2}, 1)$ point of tangency

$y - 1 = -1(x - \frac{\pi}{2})$

6. If $f'(x) = \sin x$, find $f(x)$. "Reverse"

$f(x) = -\cos x$

7. If $\frac{dy}{dx} = \frac{\csc^2 x}{2}$, find y. "Reverse"

$\frac{1}{2} \cdot \frac{\csc^2 x}{1} \qquad y = -\frac{1}{2}\cot x$

OVER →

Use the table below for problems 8-11.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	4	-1	-6	7
1	0	-6	0	5
2	-6	-5	4	3

$x=1$
$x=2 \rightarrow$

Tables and graphs ***above*** *problems are standard on the AP exam, so get used to this format . . .*

8. Find $\frac{d}{dx}(f \cdot g)$ evaluated at $x = 2$.

$$(f \cdot g)' = f' g + g' f$$
$$= -5 \cdot 4 + 3 \cdot -6$$
$$= -20 + -18$$
$$\boxed{= -38}$$

9. Find $\frac{d}{dx}\left\{\frac{f(x)}{e^x}\right\}$ evaluated at $x = 1$.

$$\left[\frac{f(x)}{e^x}\right]' = \frac{e^x \cdot f'(x) - e^x \cdot f(x)}{(e^x)^2}$$
$$= \frac{e^1 \cdot f'(1) - e^1 \cdot f(1)}{e^2}$$
$$= \frac{e \cdot -6 - e^1 \cdot 0}{e^2}$$
$$= \frac{-6e}{e^2} = \boxed{\frac{-6}{e}}$$

10. Find $\frac{d}{dx}\{\ln(x) \cdot g(x)\}$ evaluated at $x = 1$.

$$(\ln x \cdot g(x))' = \frac{1}{x} \cdot g(x) + \ln x \cdot g'(x)$$
$$= \frac{1}{1} \cdot g(1) + \ln 1 \cdot g'(1)$$
$$= 1 \cdot 0 + 0 \cdot 5$$
$$= \boxed{0}$$

11. Find $\frac{d}{dx}\left\{\frac{(g(x))^2}{\cos x}\right\}$ evaluated at $x = 0$.

$2g'(x)g(x)$

$\cancel{*}(g(x))^2 = g(x) \cdot g(x)$

$[(g(x))^2]' = g'(x) \cdot g(x) + g(x) g'(x)$

$$\left[\frac{(g(x))^2}{\cos x}\right]' = \frac{\cos x\,(2g'(x)g(x)) - (g(x))^2(-\sin x)}{(\cos x)^2}$$

When $x=0$: $$= \frac{\cos 0\,(2g'(0) \cdot g(0)) - (g(0))^2(-\sin 0)}{(\cos 0)^2}$$
$$= \frac{1\,(2 \cdot 7 \cdot -6) - (-6)^2(0)}{1}$$
$$= 14 \cdot -6 - 0$$
$$= \boxed{-84}$$